ME 305 Fluid Mechanics I

Part 8

Viscous Flow in Pipes and Ducts

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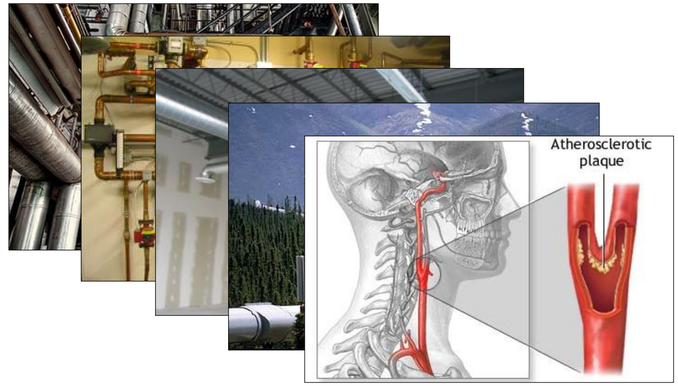
Flow in Pipes and Ducts

- Flow in closed conduits (circular pipes and non-circular ducts) are very common.

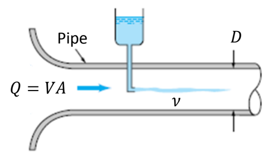


Flow in Pipes and Ducts (cont'd)

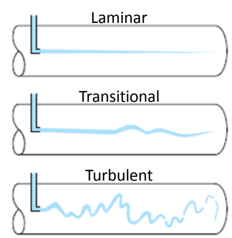
- We assume that pipes/ducts are completely filled with fluid. Other case is known as open channel flow.
- Typical systems involve pipes/ducts of various sizes connected to each other by various fittings, elbows, tees, etc.
- Valves are used to control flow rate.
- Fluid is usually forced to flow by a fan or a pump.
- We need to perform frictional head loss (pressure drop) calculations. They mostly depend on experimental results and empirical relations.
- First we need to be able to differentiate between laminar and turbulent flows.

Laminar vs. Turbulent Flows

- Laminar flow is characterized by smooth streamlines and highly ordered motion. Fluid flows as if there are immiscible layers of fluid.
- Turbulent flow is highly disordered. Usually there are unsteady, random fluctuations on top of a mean flow.
- Most flows encountered in practice are turbulent.

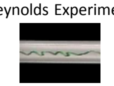


$$Re_D = \frac{VD}{\nu} = \frac{\rho VD}{\mu}$$



Movies

Reynolds Experiment



Laminar vs. Turbulent Velocity Profiles

Laminar vs. Turbulent Flows (cont'd)

- For pipe flow, transition from laminar to turbulent flow occurs at a critical Reynolds number of approximately 2300.

$$Re_D < 2300 \quad : \text{Laminar}$$

$$Re_D > 2300 \quad : \text{Turbulent}$$

Characteristic speed
Characteristic length

$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{VD}{\nu}$$

- At large Reynolds numbers inertia forces, which are proportional to $\rho V^2 D^2$, are much higher than the viscous forces. Viscous forces can not regularize random fluctuations. Fluctuations grow and the flow becomes turbulent.
- At small Reynolds numbers viscous forces are high enough to suppress random fluctuations and keep the flow in order, i.e. laminar.

Exercise : Estimate the typical Reynolds number of the flow inside the pipe that supplies water to your shower.

Pressure Loss (Pressure Drop, Head Loss) in Pipe Flow

- As a fluid flows in a straight, constant diameter pipe its pressure drops due to viscous effects, known as major pressure loss.
- Additional pressure drops occur due to other components such as valves, bends, tees, sudden expansions, sudden contractions, etc. These are known as minor pressure losses.
- In Chapter 6 we studied analytical solution of Hagen-Poiseuille flow, which is the steady, fully-developed, laminar flow in a circular pipe.
- We showed that $\frac{dp}{dx}$ = constant and pressure drop over a pipe section of length L is

Average speed

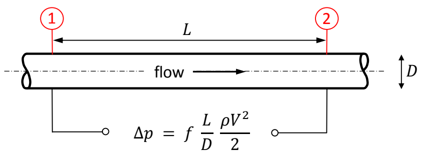
$$\Delta p = f \frac{L}{D} \frac{\rho V^2}{2}$$

Darcy friction factor

Dynamic pressure

where $f = 64/Re_D$ for laminar flow, with $Re_D = \frac{VD}{\nu}$.

Major Pressure Loss in Laminar Pipe Flow



- Consider the EBE between sections 1 and 2.

$$\frac{p_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \frac{p_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + h_f$$

- Friction head is related to the pressure drop as

$$\boxed{h_f = \frac{\Delta p}{\rho g} = f \frac{L}{D} \frac{V^2}{2g}}$$

- The above important equation is known as the Darcy-Weisbach equation.

Major Pressure Loss in Turbulent Pipe Flow

- Analytical solution of pipe flow is available only for laminar flows.
- It is not possible to solve Navier-Stokes equations analytically for turbulent flows.
- Instead we need to determine pressure drop values experimentally.
- For a turbulent pipe flow pressure drop depends on

$$\Delta p = \Delta p\,(L, D, V, \mu, \rho, \varepsilon)$$

New parameter:
Surface roughness

- A Buckingham-Pi analysis yields the following relation

$$\frac{\Delta p}{\rho V^2/2} = F\left(\frac{L}{D}, Re_D, \frac{\varepsilon}{D}\right)$$

which can be expressed in the same way as laminar flow $\Delta p = f\left(Re_D, \frac{\varepsilon}{D}\right) \frac{L}{D} \frac{\rho V^2}{2}$

Major Pressure Loss in Turbulent Pipe Flow (cont'd)

- For turbulent pipe flows friction factor is a function of Reynolds number and relative surface roughness.

$$f\left(Re_D, \frac{\varepsilon}{D}\right)$$

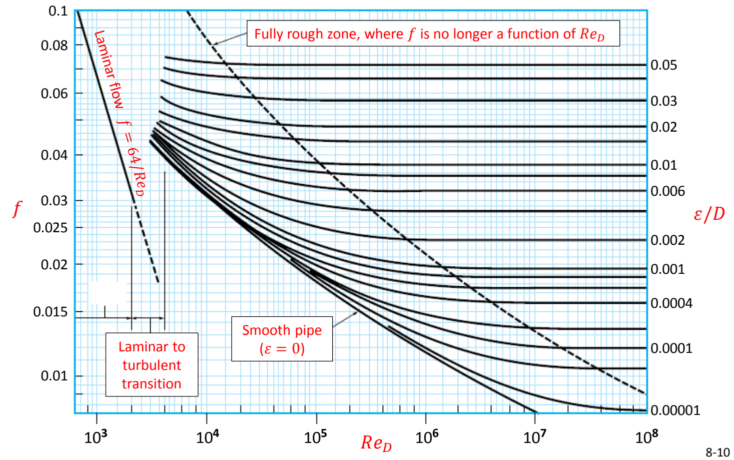
Relative surface roughness

- In 1933 Nikuradse performed very detailed experiments to determine friction factor for turbulent flows.
- Later Colebrook presented this experimental data as the following equation

$$\text{Colebrook Formula}: \quad \frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re_D\sqrt{f}}\right)$$

- An easy to use graphical representation of this equation is known as the Moody diagram, given in the next slide).
- Be careful in reading the logarithmic horizontal and vertical axes of the Moody diagram.

Moody Diagram to Read Friction Factor



Moody Diagram (cont'd)

- To use the Moody diagram first we read the roughness of the pipe of interest from a reference (see the next slide).
- Next we calculate the relative roughness (ε/D) and Reynolds number, and read the friction factor value f.
- $f = 64/Re_D$ relation for laminar flows appears in the Moody diagram as a straight line.
- Even for smooth pipes ($\varepsilon = 0$) friction factor is not zero, as expected.
- For high Re_D values f becomes independent of Re_D. This region is known as fully rough (or fully turbulent) flow.
- Between the laminar and turbulent regions, there is a transition region ($2100 < Re_D < 4000$), where the experimental data is not very reliable.
- We do not expect to read f with more than 10 % accuracy from the Moody diagram.

Moody Diagram (cont'd)

	Roughness, ε
Pipe	**Millimeters**
Riveted steel	0.9–9.0
Concrete	0.3–3.0
Wood stave	0.18–0.9
Cast iron	0.26
Galvanized iron	0.15
Commercial steel or wrought iron	0.045
Drawn tubing	0.0015
Plastic, glass	0.0 (smooth)

For pipes which are in service for a long time, roughness values given in tables for new pipes should be used with caution.

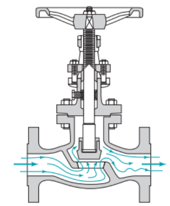
- As an alternative to the Moody diagram, Haaland's equation (explicit and easy to use) can be used to calculate f.

$$\frac{1}{\sqrt{f}} = -1.8\log\left(\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{Re_D}\right)$$

Example

Exercise : Crude oil flows through a level section of a pipeline at a rate of 2.944 m^3/s. The pipe inside diameter is 1.22 m, its roughness is equivalent to galvanized iron. The maximum allowable pressure is 8.27 MPa. The minimum pressure required to keep dissolved gases in solution in the crude oil is 344.5 kPa. The crude oil has a specific gravity of 0.93. Viscosity at the pumping temperature of 60 ℃ is μ = 0.0168 Pa s. a) Determine the maximum allowable spacing between the pumping stations. b) If the pump efficiency is 85%, determine the power that must be supplied at each umping station. (Reference: Fox's book)

Minor Head Losses



- Pressure drops due to the flow through valves, bends, tees, sudden area changes, etc. are known as minor losses.
- Minor head losses can be calculated as

$$h_f = k\frac{V^2}{2g}$$

Head loss coefficient

- For certain piping system elements minor losses are given in terms of an equivalent length

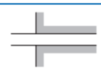
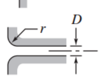
$$h_f = f\frac{L_e}{D}\frac{V^2}{2g}$$

where L_e is the length of a straight pipe section that would create a major loss equal to the minor loss created by the element. f is obtained from the Moody diagram.

- k and L_e values are obtained from a figure or table, such as the ones given in the coming slides.

Minor Head Losses (cont'd)

Minor Loss Coefficients for Pipe Entrances

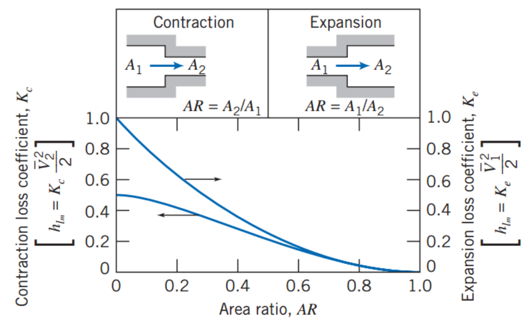
Entrance Type		Minor Loss Coefficient, K^a
Reentrant		0.78
Square-edged		0.5
Rounded		r/D: 0.02, 0.06, ≥ 0.15; K: 0.28, 0.15, 0.04

[a]Based on $h_{l_m} = K(\bar{V}^2/2)$, where $\bar{V}$ is the mean velocity in the pipe.

Fox's book

Minor Head Losses (cont'd)

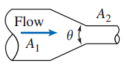
Loss coefficients for flow through sudden area changes



Fox's book

Minor Head Losses (cont'd)

Loss Coefficients (K) for Gradual Contractions: Round and Rectangular Ducts



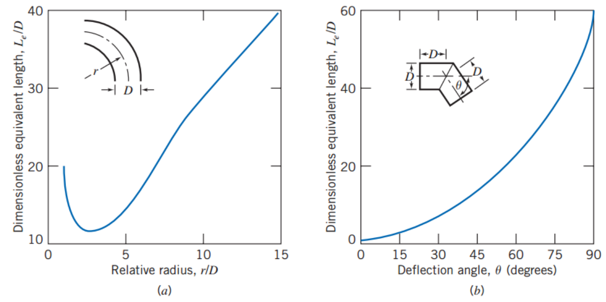
	Included Angle, θ, Degrees						
A_2/A_1	10	15–40	50–60	90	120	150	180
0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

Note: Coefficients are based on $h_{l_m} = K(\bar{V}_2^2/2)$.

Fox's book

Minor Head Losses (cont'd)

Representative total resistance (L_e/D) for (a) 90° pipe bends and flanged elbows, and (b) miter bends



Fox's book

Minor Head Losses (cont'd)

Representative Dimensionless Equivalent Lengths (L_e/D) for Valves and Fittings

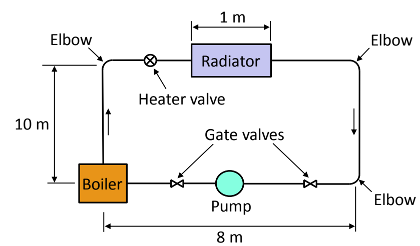
Fitting Type	Equivalent Length,[a] L_e/D
Valves (fully open)	
Gate valve	8
Globe valve	340
Angle valve	150
Ball valve	3
Lift check valve: globe lift	600
angle lift	55
Foot valve with strainer: poppet disk	420
hinged disk	75
Standard elbow: 90°	30
45°	16
Return bend, close pattern	50
Standard tee: flow through run	20
flow through branch	60

[a]Based on $h_{l_m} = f(L_e/D)(\bar{V}^2/2)$.

Fox's book

Examples (cont'd)

Exercise : Sketch of the warm water heating system of a studio apartment is shown in the figure. Volumetric flow rate of water through the piping system with a diameter of 1.25 cm is 0.2 L/s. Friction factor is 0.02. Head loss coefficients for the elbow, heater valve, radiator, gate valve and boiler are 2.0, 5.0, 3.0, 1.0 and 3.0, respectively. Determine the power required to drive the circulation pump, which has an efficiency of 75%.



Examples (cont’d)

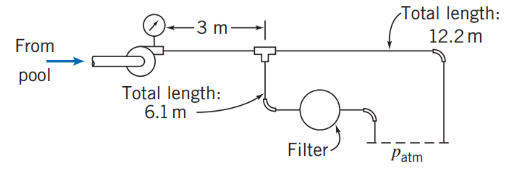
- Certain problems require an iterative solution because Re_D and f cannot be computed directly. Solution starts with an initial guess for f and/or D. Usually couple of iterations are enough to get the final answer.
- Typical problems of this sort are
  - L and D of the pipe are known. Find Q for a specified Δp.
  - L of the pipe is known. Find D for a specified Q and Δp.
  - Multiple path systems

Exercise : Air at 1 atm and 35 °C is to be transported in a 150 m long plastic pipe at a rate of 0.35 $\mathrm{m^3/s}$. If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.

Exercise : Reconsider the previous example. Now the duct length is doubled while its diameter is maintained constant. If the total head loss is to remain constant, determine the new flow rate through the duct.

Examples (cont’d)

Exercise : A swimming pool has a partial-flow filtration system. Water at 24 °C is pumped from the pool through the system shown. The pump delivers 1.9 L/s. The pipe is nominal 20 mm PVC (internal diameter 20.93 mm). The pressure loss through the filter is approximately $\Delta p = 1039\ Q^2$, where Δp is in kPa and Q is in L/s. Determine the pump pressure and the flow rate through each branch of the system (Reference: Fox’s book).



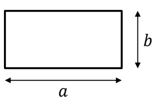
Non-Circular Geometries

- Correlations derived for circular pipes can be used for noncircular ones, provided that the cross sections are not too exaggerated.
- Ducts with square or rectangular cross sections can be treated if their height to width ratio is less than about 3 or 4.
- To do this we use the hydraulic diameter concept

$$D_h = \frac{4A}{P}$$

Cross sectional area (A)

Wetted perimeter (length of wall in contact with fluid at any cross-section) (P)



$$D_h = \frac{4(ab)}{2(a+b)}$$

is equivalent to